Honeycomb Toll Plaza

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Abstract

We analyze the performance of a common toll plaza design compared to our proposed new improved toll plaza. The new design would reduce the cost, decrease the probability of collision at merge points, and increase throughput.

Our proposed cellular toll plaza resembles a honeycomb. Each hexagon contains two toll booths, which serve two separated vehicle streams that are merged in advance before they re-enter the highway. The total area of the proposed plaza is reduced significantly. Also, the average waiting time in queue is diminished, which means that throughput is increased . Additionally, by splitting the merging into two stages, the probability of accidents is decreased.

Our main contributions are:

- The new cellular architecture can greatly reduce the construction area compared with traditional linear distributed toll booths.
- We analyze the throughput of toll plazas by means of queuing theory. To verify our theory, we simulate the behavior of a large number of vehicles passing the toll plaza, with the help of PTV-VISSIM. Simulation results show that our cellular toll plazas have better results than traditional toll plazas, especially when the traffic flow is heavy; the average travel time is reduced by about 55% and the average delay time is reduced by about 70%.
- We analyze the influence of the proportions of varied types of toll booths to our design. According to sources, the impact of exact-change toll booths is similar to manual toll booths; so we consider only two kinds of toll booths: human-staffed (MTC) toll booths and E-ZPass (ETC) toll booths. Simulation results using PTV-VISSIM show that an ETC toll booth is 8 times as fast as an MTC toll booth.

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- We simulate the performance of the cellular toll plaza under varied traffic throughput. Simulation results show that the average transit time remains at about 11 s under throughputs up to 2,000 vehicles/hr). We conclude that our model is not sensitive to traffic flow variations and has strong robustness.
- To further reduce the probability of accidents, we make the transition zone smoother and rearrange the different kinds of toll booths.
- For self-driving vehicles, we reserve special E-ZPass toll booths in the center of the toll plaza, which match the characteristics of autonomous vehicles: safer and faster.

Electronic toll collection and autonomous vehicles are the trends of modern transportation. Our new design can improve the performance of toll collection in terms of cost, throughput, and accident prevention.

Introduction

Problem Background

Research found that 36% of total car travel time in China is delay time caused by tolling [1]. In addition, as a vehicle-intensive place, the toll plaza has become an accident-prone section [2]. The congestion problem at toll plazas becomes more and more serious due to the outdated design.

With the widespread use of Electronic Toll Collection (ETC) (such as the E-ZPass system in the U.S.) to replace Manual Toll Collection (MTC), the efficiency of toll collection has improved significantly and relieved some congestion at the toll plaza. However, due to the higher speed of vehicles passing through the toll plaza, the probability of collision in the merging zone is increased.

We design a toll plaza based on bionics: a honeycomb hexagonal tiling creates equal-sized cells while minimizing the total perimeter of the cells [3]. Such cellular structure is widely used; for example, the base stations of mobile communications are distributed in honeycomb-like fashion. In our toll plaza, the toll booths are located in regular hexagons.

Description of Terminology

- **Total time cost:** The average time interval for a vehicle from the beginning point of the detection area to the ending point of the detection area.
- **Theoretical time cost:** If there is only one vehicle in the system and that vehicle is not limited by a control signal, the time interval for that vehicle from the beginning point of the detection area to the ending point of the detection area is the theoretical time cost.

- **Time delayed:** Total time cost minus theoretical time cost.
- *L*: the number of lanes in each direction of the highway.
- *B*: the total number of toll booths in each direction.

Our Work

With the popularization of ETC equipment and autonomous vehicles, MTC lanes will be totally replaced by ETC lanes in the next 20 years; that will increase the road capacity and decrease the time cost for each car.

Present toll plazas of traditional design occupy a large area, and the cost of construction is high. With an increase in vehicles' speed, congestion at the merge points may increase the probability of accidents.

- We design a model of a cellular toll plaza, including its shape, size, and merge.
- We quantify how much we could reduce the area of the toll plaza.
- We use the VISSIM software to simulate a cellular toll plaza.
- We use VISSIM to analyze the traffic capacity of a cellular toll plaza with a mixture of toll lanes (MTC, ETC, etc.), for both heavy traffic and light traffic.
- Based on our results, we improve our design in three particular aspects.
- We analyze the effect of traffic flow on the capacity of the toll plaza.
- We investigate whether the toll plaza can meet the needs of autonomous vehicles.

Figure 1 shows the process of evolution of our design.

General Assumptions

- The arrival of vehicles follows a Poisson distribution.
- In general, the traffic at ETC toll booths should be much heavier than the traffic at other types of toll booth.
- All toll booths are ETC or E-ZPass unless otherwise specified.
- There are no entrance or exit ramps near the toll booths.
- The service procedure at the toll booths, and the merging procedure of the vehicles after the toll booths, first-come, first-served queuing systems.



Figure 1. Evolution of our design.

Design of a Honeycomb-Like Toll Plaza

In traditional toll plazas, there are more toll booths than lanes of incoming traffic. A toll plaza consists of

- the fan-out area before the toll booths,
- the toll booths themselves, and
- the fan-in area after the toll booths.

The toll booths are often constructed in a straight line across the highway, perpendicular to the direction of traffic flow. Therefore, the area of the toll plaza is large. To reduce the area and save on the construction cost, we design a new kind of toll plaza based on the structure of the honeycomb. In addition, by splitting the merging procedure into two stages, our new design can reduce the probability of collision, in contrast to the traditional merging procedure, where a large number of vehicles concentrate onto the highway simultaneously. The evolution of our design is shown in **Figure 2**, where we smooth the transition zone to avoid sharp turns and add some reserved toll booths in the middle for autonomous vehicles.



Figure 2. Evolution of our design.

Model Design

Estimated Cost of the Toll Plaza

The main cost of building a toll plaza includes the construction costs of the road surface and of the toll booths. We assess the land area and try to minimize it. The toll plaza's total area can be divided into zones as shown in **Figure 3**.



Figure 3. Design and design parameters for a cellular toll plaza.

We establish some parameters:

- n_t = number of toll booths,
- w_t = width of a toll booth,
- n_l = number of the lanes of the highway,
- w_l = width of a lane,
- w_o = tangential offset width,
- l_t = length of the transition zone, and
- v = design speed.

Comparison of the Areas of the Toll Plazas

[EDITOR'S NOTE: We omit the authors' calculation of the geometric areas of the traditional toll plaza and of the cellular toll plaza. **Figure 4** serves well to illustrate their point.]

The cellular toll plaza can significantly save space compared with the traditional toll plaza. The effect can be seen in **Figure 4**.



Figure 4. Comparison of the footprints of the cellular toll plaza and the traditional toll plaza.

Analysis of the Throughput of Toll Plazas

We consider the entire process of tolling as the operation of two seriallyconnected queuing systems:

- vehicles passing through the toll booths and queuing in front of the toll booths, and
- vehicles passing through the merge points at the exit of the toll plaza.

Queuing System at Toll Collectors

When a vehicle enters the toll plaza, the driver will head to a toll booth according to certain principles, such as the distance to each toll booth and the number of vehicles waiting in each queue. In our model:

• The arrival of vehicles at each toll booth is a Poisson process, so the interval between vehicles arriving at a toll booth follows an exponential distribution.

• The time cost for each vehicle at the toll booth also follows an exponential distribution.

Each toll booth can handle only one lane at each time. In our model there are two toll booths, on each toll island, one on each side, each serving a single lane.

In summary, we believe that each toll booth can be considered an M/M/1 queuing system, meaning that the time between arrivals is exponential ("M" for "Markovian"), the service time is exponential, and there is a single server.

Queuing at Merge Points

We note Burke's theorem:

If the arrival time and service time of a M/M/1 queuing model is a Poisson process with parameter λ , the departure process of the queuing model is also a Poisson process with parameter λ [5].

Since the output of a toll booth is a Poisson process, arrivals at the merge points are also Poisson processes.

Current U.S. road design guidelines stipulate that lane merging can only be started from the right side of the vehicle's driving direction and only one lane can be merged at a time [6]. According to this provision, and also to simplify the model, we divide the lanes into two types:

- A Type I lane doesn't pass through any merge point, but
- a Type II lane does.

Recall that there are *L* highway lanes and *B* toll booths; Type II lanes all merge into a single highway lane, so there are (L - 1) Type I lanes and (B - L + 1) Type II lanes. Type II lanes need to merge with all lanes on their right; for example, in **Figure 6**, lane 4 needs to merge with cars from lanes 1, 2, and 3, while lane 5 is of Type I and does no merging.



Figure 6. Merge points in the transition zone.

For vehicles in Type I lanes, which can pass through without merging, we consider the total time cost to be the number of merge points times the

time cost for passing each of them, that is,

$$\begin{cases} (B-L) \times \frac{1}{\mu_0}, & \text{ for the traditional toll plaza; and} \\ \left(\frac{B}{2} - L\right) \times \frac{1}{\mu_0}, & \text{ for the cellular toll plaza,} \end{cases}$$

where μ_0 is the service rate at a merge point when there is no merge conflict.

The probability of a vehicle being in a Type I lane is (L-1)/B for the traditional toll plaza and (L-1)/(B/2) for our cellular toll plaza.

The probability of a vehicle being in a Type II lane is (B - L + 1)/B for the traditional toll plaza and (B - 2L + 2)/B for our cellular toll plaza. For a vehicle in a Type II lane, we must take into account not only the number of merge points that it must pass through but also the probability of merging at a merge point.

At the *k*th merge point, the probability of arrival of two vehicles simultaneously is the sum of the probability of one lane plus the probability of a vehicle from the (k - 1)st merge point, that is, (k + 1)/B and 2(k + 1)/B, respectively. Take the merging pattern shown in **Figure 6** as an example, the probability of merging at merge point 1 equals the probability of a vehicle in lane 1 plus the probability of a vehicle in lane 2, that is, 1/B + 1/B = 2/B; similarly, at merge point 2, the probability is the probability of a vehicle in lane 3 plus the probability of a vehicle in merge point 1, that is, 1/B + 2/B = 3/B. The traffic flow at merge point *k* is thus $(k + 1)\Phi/B$, where Φ is the total traffic flow.

To simplify the model, we don't distinguish between the two lanes that merge at the same merge point; that is, the time for a vehicle to pass through a single merge point is independent of which of the two lanes it is in. If there is no vehicle in the other lane, or a vehicle in the other lane that slows or stops to avoid the merging conflict, this vehicle can complete the merging process without deceleration, that is, with time cost $1/\mu_0$. Otherwise, this vehicle needs to slow or stop and wait, with time cost defined as $1/\mu_1$.

In summary, the arrival rate of the queuing system at merge points follows an exponential distribution (i.e., it is a Poisson process), but the service rate is a more general function; that is, we have an M/G/1 queuing model (with "G" for "General").

Calculation

Parameter Assignment

- *B*: Number of toll booths In reality, the number of toll booths depends on the traffic flow, types of vehicles, etc. However, to simplify our model, we set B = 8.
- *L*: Number of lanes of highway We set L = 3.

- μ_T: Service rate of a toll booth We take the service rate to be 1200 vehicles/hr, corresponding to that of a widely-installed current electronic toll system [7].
- μ₀: Service rate at a merge point when no merging conflict occurs, or conflict occurs but the other vehicle slows or stops to wait We also take μ₀ as the service rate when a vehicle directly heads for a lane of the highway without passing any merge point. The average speed of vehicles on the highway is 60 mph [7]. Therefore, the length of the merge point should be the length of a normal vehicle (15 ft) plus a safe distance, which is six times the length of a normal vehicle [7]; so the length of the merge point is 105 ft. Then the average time for a vehicle to pass through a merge point is 105 ft / 60 mph ≈ 1.2 s. We take the reciprocal of this as the value of μ₀, converted to an hourly basis:

$$\mu_0 = \frac{3600 \text{ s}}{\text{hr}} \times \frac{1}{1.2 \text{ s}} \approx 3017 \text{ vehicles/hr.}$$

• μ_1 : Service rate at a merge point when a merging conflict occurs This rate applies to a vehicle that stops to avoid another vehicle when two vehicles reach the merge point at the same time. When starting again, the vehicle will start at 0 m/s, and the (additional) vehicle safe distance is only one times the length of the vehicle [7]. From the displacement formula $s = \frac{1}{2}at^2$, we can derive $t = \sqrt{2s/a}$. The average acceleration of such a vehicle is 6.5 ft/s² [1]. Substituting, we find

$$\sqrt{\frac{15 \text{ ft} + 15 \text{ ft}}{6.5 \text{ ft/s}^2}} \approx 3.0 \text{ s.}$$

As before, we take the reciprocal and convert to an hourly basis, getting $\mu_1 = 1185$ vehicles/hr.

Time Cost at Toll Booths for the Cellular Model

Based on the arrival rate of a lane as given above, we can calculate the average time spent by a vehicle at the toll booth according to the formula below given in [8]. The formula applies to both the traditional toll plaza and our new design, since in the cellular toll plaza, each toll booth faces the same traffic flow as one lane of the traditional toll plaza.

$$W_T = \frac{1}{\mu_T - \frac{\Phi}{B}},$$

where

• W_T is the time cost passing through a toll booth,

- μ_T is the service rate of a toll booth,
- Φ is the traffic flow, and
- *B* is the number of toll booths.

Time Cost at Merge Points for the Cellular Model

The merge process at each merge point is essentially a birth-death process. **Figure 7** describes the state transition of this process in the form of a Markov chain.



Figure 7. State transitions of a birth-death process.

In this process, each state follows the rule that the sum of the transit-in probabilities equals to the sum of the transit-out probabilities [5], and the probability sum of all events is 1. We let P_i be the probability of being in state *i*. Then we have the equations below:

$$\lambda P_0 = \mu_0 P_1;$$

$$\lambda P_1 + \mu_0 P_1 = \lambda P_0 + \mu_1 P_2;$$

$$\lambda P_n + \mu_0 P_n = \lambda P_{n-1} + \mu_1 P_{n+1}, \qquad n \ge 2;$$

$$\sum_{i=0}^{\infty} P_i = 1,$$

where our interpretations in the toll plaza setting are:

- P_n is the probability of n vehicles in the system,
- λ is the arrival rate at a merge point,
- μ_0 is the service rate at a merge point when there is no merging conflict,
- μ_1 is the service rate at a merge point when merging conflict occurs.

Solving the equations above for the values of the P_i , we obtain:

$$P_{0} = \left(1 + \frac{\lambda}{\mu_{0}} + \frac{2\lambda\mu_{1}}{\mu_{0}^{2}\mu_{1} - \lambda\mu_{0}^{2} + \lambda\mu_{0}\mu_{1} - \lambda^{2}\mu_{0}}\right)^{-1}$$

$$P_{1} = \frac{\lambda}{\mu_{0}}P_{0},$$

$$P_{n} = \frac{2\lambda^{2}}{\mu_{0}^{2} + \lambda\mu_{0}} \left(\frac{\lambda}{\mu_{1}}\right)^{n-2}P_{0}, \quad n \ge 2.$$

From the probabilities obtained above, we can calculate the expected number of vehicles L_s in the whole queuing system:

$$L_s(\lambda) = \sum_{i=1}^{\infty} iP_i = \frac{\lambda}{\mu_1 - \lambda} + \frac{\lambda\mu_1 - \lambda\mu_0}{\lambda\mu_1 - \lambda\mu_0 + \mu_0\mu_1}$$

 L_s is also called the average queue length. According to Little's Law [5], the average waiting time W_s for a vehicle at a merge point is

$$W_s = \frac{L_s}{\lambda}.$$

Total Time Costs for the Two Models

According to our assumptions and calculations, the traffic flow at the kth merge point in a traditional toll plaza is

$$\frac{(k+1)\Phi}{B}, \quad k = 1, \dots, (B-L+1).$$

Hence the probability of arrival of a vehicle at the *k*th merge point is (k + 1)/B.

According to the formulas above, the total average time cost at the merge point is

$$W_{MT} = \frac{L-1}{B} \cdot \frac{B-L}{\mu_0} + \frac{B-L+1}{B} \sum_{k=1}^{B-L} \frac{k+1}{B} W_s \left(\frac{k+1}{B} \Phi\right).$$

Adding the time cost passing through each toll booth obtained above, we can calculate the average time cost passing through the whole *traditional toll plaza*:

$$W_{AT} = W_T + W_{MT}$$

= $\frac{1}{\mu_T - \frac{\Phi}{B}} + \frac{L-1}{B} \cdot \frac{B-L}{\mu_0} + \frac{B-L+1}{B} \sum_{k=1}^{B-L} \frac{k+1}{B} W_s \left(\frac{k+1}{B}\Phi\right)$

But in our design, since the traffic flow merges in advance, the traffic flow of each lane becomes twice that of the previous lane, and the number of lanes is reduced by half. To simplify the calculation, we may assume that B is always even, so that the traffic flow at the kth merge point is

$$\frac{2(k+1)\Phi}{B}$$
, $k = 1, \dots, \left(\frac{B}{2} - L + 1\right)$,

so the probability of an arrival at the *k*th merge point is

$$\frac{2(k+1)}{B}$$
, $k = 1, \dots, \left(\frac{B}{2} - L + 1\right)$.

The total average time spent by a vehicle at merge points in the cellular toll plaza is is

$$W_{MI} = \frac{2(L-1)}{B} \cdot \frac{B-2L}{2\mu_0} + \frac{B-2L+2}{B} \sum_{k=1}^{\frac{B}{2}-L} \frac{2(k+1)}{B} W_s \left(\frac{2(k+1)}{B} \Phi\right)$$

In a cellular toll plaza, all lanes are merged in advance. So we need to calculate the additional time cost of pre-merging process:

$$W_{Ex} = W_s \left(\frac{2\Phi}{B}\right).$$

Adding together all the time costs, we find that the average time cost for each vehicle passing through the *cellular toll plaza* is

$$\begin{split} W_{AI} &= W_T + W_{MI} + W_{Ex} \\ &= \frac{1}{\mu_T - \frac{\Phi}{B}} + \frac{L - 1}{B} \cdot \frac{B - L}{\mu_0} + \frac{B - L + 1}{B} \sum_{k=1}^{B - L} \frac{k + 1}{B} W_s \left(\frac{k + 1}{B} \Phi\right) \\ &+ W_s \left(\frac{2\Phi}{B}\right). \end{split}$$

Substituting the specific values of the parameters and plotting, we see the comparison results shown in **Figure 8**.

Improve the Accident Prevention Ability

Hierarchical Merge Pattern

A toll booth in a cellular toll plaza has only one merge point at the end of the transition zone, increasing the possibility of it being overcrowded. However, in our cellular toll plaza, there are bends among the cells, keeping the speed of vehicles within a safe range. Therefore, we decrease the possibility of accidents caused by traffic congestion as well as those caused by excessive speed.

Figure 9 and Figure 10 illustrate the contrast.



Figure 8. Comparison of total time cost to pass through a traditional toll plaza vs. through a cellular toll plaza, for different values of traffic flow.



Figure 9. Merging in a cellular toll plaza: low congestion, low speed.



Figure 10. Merging in a traditional toll plaza: high possibility of conflict, potentially higher speed.

A Gentle Design of the Transition Zone

The gradient of the transition zone is set according to the design speed and the tangential offset width; different countries have different standards: The maximum ratio of the U.S. standard is 1:20, and the minimum is 1:8 [9].

Therefore, we further improve the model to improve safety, by changing the gradient rate of cross-section of the toll plaza, as shown in **Figure 11**.



Figure 11. Improved design of the cellular toll plaza.

More Suitable for ETC Technology

Toll plazas typically include a different set of charging models: conventional (human-staffed) toll booths, exact-change (automated) toll booths, and electronic toll collection (ETC) toll booths. Vehicles near the entrance to the toll plaza often encounter traffic accidents due to the choice of different access routes; so the locations of the different types of toll booths is also critical to safety.

With ETC technology, the ways vehicles enter, drive through, and exit the toll plaza are different from the traditional charging pattern. A computational experiment showed that booths associated to higher-risk traffic flows—e.g., traffic such as that directed to ETC toll booths, which approaches the toll plaza at higher speeds—should be located in a central position with respect to other booth types [10]. So, we add two ETC lanes in the middle of the new toll plaza (see **Figure 12**).



Figure 12. Further improved design of the cellular toll plaza.

The Influence of Autonomous Vehicles

Compared to the traditional toll plaza, a cellular toll plaza can better meet the needs of autonomous ("smart") cars, since they do not pay in cash

We first analyze the principle and characteristics of autonomous vehicles, then optimize the cellular toll plaza model.

Characteristics of Autonomous Vehicles

- Autonomous cars need to be equipped with an automatic payment system, that is, ETC equipment, which means that they can pass through the toll plaza quickly.
- Self-driving vehicles have better control, which can reduce the possibility of accidents in the toll plaza. Besides, a driver's own factors (such as bad mood, disputes with the toll plaza service staff, etc.) will not affect the vehicle's safety of an autonomous vehicle [11].
- Vehicles at the junction of the toll plaza convergence can be more orderly, thus avoiding congestion and maximizing the efficiency of cellular toll plazas.

Our Solution

- Since in the future autonomous vehicles will be more numerous, to maximize efficiency, cellular toll plazas must increase the number of automatic toll booths and reduce the number of manual toll booths. The remaining MTC lanes would be located at the sides of the toll plaza, with straight lanes for large vehicles set in the middle. Since autonomous vehicles are easy to steer, the remaining lanes would all be ETC lanes.
- According to the queuing model and the simulation result from VISSIM, the throughput of the toll plaza increases with a greater proportion of ETC lanes. Since autonomous vehicles are all non-cash payment, when compared with the traditional toll plaza, the cellular design is more suitable.

Analysis of our Design

Using Simulation

Basic Data for the Simulation

We use PTV-VISSIM 4.3 to do this simulation [12]. We set the speed of the vehicle through the ETC deceleration belt at 24 km/h [13] and deceleration is 2 m/s^2 .

We did two pairs of simulations comparing the cellular toll plaza to a traditional one. Both sets had 3 highway lanes and 8 toll booths. The difference was in the number of ETC lanes: 8 in the first simulation, and 2 in the second.

We show the results of the simulations in **Figure 13** and **Figure 14**. Here we address some questions that may arise in the mind of the reader:

• Q: Why consider ETC tolling but not exact-change toll booths?



Figure 13. Total time cost vs. traffic flow, for traditional toll plaza (solid curve) and cellular toll plaza (dotted curve) with 8 lanes, all ETC.



Figure 14. Total time cost vs. traffic flow, for traditional toll plaza (solid curve) and cellular toll plaza (dotted curve) with 2 ETC and 6 MTC lanes.

- A: If we considered all possible charging patterns, our model would be too complicated. According to [14], there is little difference between the performance of MTC toll booths and that of exact-change toll booths: 425 vehicles/hr vs. 500 vehicles/hr. So we can combine the MTC method and the exact-change method into a single "cash" method. ETC systems can achieve between 1,200 vehicles/hr and 1,800 vehicles/hr.
- Q: Why not consider autonomous vehicles?
 - A: Because autonomous vehicles don't need a driver. Just install an ETC device on the car.
- Q: How to explain the great difference between the result of the queuing model and the result of the VISSIM simulation?
 - A: The VISSIM software has taken a lot of factors into consideration. Therefore, compared with the pure theoretical derivation, VISSIM is more practical.
- Q: How to explain the great change at a traffic flow of 2,000 vehicles/hr.
 - A: Both kinds of toll plaza have a maximum capacity and throughput.

Simulation Conclusions

- All lanes ETC (Figure 13) When the toll plaza is configured to all lanes ETC, the traditional toll plaza and the cellular plaza have about the same throughput for light traffic. But the cellular plaza is better than the traditional plaza in heavy traffic (over 2,200 vehicles/hr), with less than half as much total time cost. The simulation results are in good agreement with the results of queuing theory.
- 2 ETC lanes, 6 MTC lanes (Figure 14) The cellular plaza produces less total time delay for traffic flows between 400 vehicles/hr and 900 vehicles/hr.

The Influence of Different Kinds of Toll Booths

Results of the Analysis

Figure 15 shows a comparison of total time cost (and also time delayed) for 0 through 8 ETC lanes (and correspondingly 8 through 0 cash lanes), for a traffic flow of 2,400 vehicles/hr.

The capacity of a cellular toll plaza is more sensitive to the proportion of ETC lanes than a traditional toll plaza. The smaller the proportion of cash lanes, the shorter the average transit time and average delay.

With 8 ETC lanes, all cars pass through without any time delay. Moreover, we have:

The Rule of 8: 8 ETC lanes are 8 times as fast as 8 cash lanes.



Figure 15. Total time cost and time delayed vs. number of ETC lanes (out of 8), for a cellular toll plaza with 8 lanes.

Strengths and Weaknesses

Strengths

- Cellular toll plazas would save land area and reduce construction costs.
- The pre-merging in cellular toll plazas would prevent congestion.
- Cellular toll plazas would force deceleration and slower speeds through the toll plaza, preventing accidents caused by the speed difference between ETC lanes and MTC lanes.
- Our simulations confirm results of our queuing model. Simulation results show that a cellular toll plaza with ETC lanes would have better results than a traditional toll plaza, especially when traffic is heavy.
- Our simulations produced results not directly obtainable from the queuing model, for combinations of ETC and MTC.

Weaknesses

- The length of vehicles should be taken into consideration, since some long vehicles could not pass through the cellular toll plaza.
- Because actual data on traffic composition, speed, and acceleration are not easy to obtain, the traffic simulations may not be close enough to reality. We had to consult a large number of references to determine the average speed of a vehicle passing through a toll plaza, and we then needed to simulate its progress through the deceleration zone.

• In the simulations, we ignore the difference between exact-change lanes and MTC lanes.

Conclusion

With the popularization of ETC equipment and autonomous vehicles, MTC lanes will be totally replaced by ETC lanes in the next 20 years. That change will increase the road capacity and decrease the time cost for each car.

A traditional toll plaza covers a large area, and the cost of construction is high. There is congestion at merge points, which costs time and can lead to accidents.

To solve these problems, we put forward a new design, that of a cellular toll plaza, inspired by the honeycomb. Toll booths are located on hexagons. Thanks to this special structure, the cost of construction can be significantly reduced. Meanwhile, by pre-merging the traffic inside the toll plaza, the expected time cost for vehicles in merging can be greatly diminished. In addition, by appropriate design, the probability of collision at merge points can be decreased.

References

- [1] Wang, Dianhai. 2002. *Traffic Flow Theory*. Beijing: China Communications Press.
- [2] Wu, Xiaowu. 2004. Study on traffic safety of toll station of expressway. Chang'an University.
- [3] Wikipedia. 2017. Honeycomb. https://en.wikipedia.org/wiki/ Honeycomb.
- [4] Wikipedia. 2017. Queueing theory. https://en.wikipedia.org/ wiki/Queueing_theory.
- [5] Gross, D. 2008. *Fundamentals of Queueing Theory*. New York: John Wiley & Sons.
- [6] Washington State Department of Transportation. 2017. Design Manual, Chapter 1210, Geometric Plan Elements. http://www.wsdot.wa.gov/ publications/manuals/fulltext/M22-01/1210.pdf.
- [7] Transportation Research Board. 2017. *Highway Capacity Manual*. 6th ed. Washington, DC: Transportation Research Board.
- [8] Hock, Ng Chee. 1996. *Queueing Modeling Fundamentals*. New York: Wiley.

- [9] CHENG, Jianxing, and PING Jiang. 1995. Design of toll collection station for Quanzhou to Xiamen freeway. *Journal of Highway & Transportation Research and Development*.
- [10] Pratelli A., and F. Schoen. 2006. Multi-toll-type motorway stations optimal layout. In Urban Transport XII: Urban Transport and the Environment in the 21st Century, 911–921. https://www.researchgate.net/profile/Antonio_Pratelli/ publication/271450547_Multi-toll-type_motorway_stations_ optimal_layout/links/574308e408ae9ace8418becd.pdf.
- [11] Sivak, Michael, and Brandon Schoettle. 2015. Road safety with selfdriving vehicles: General limitations and road sharing with conventional vehicles. https://deepblue.lib.umich.edu/bitstream/ handle/2027.42/111735/103187.pdf?sequ.
- [12] PTV Group. n.d. Vissim. http://vision-traffic.ptvgroup.com/ en-us/products/ptv-vissim/.
- [13] Liu, Lili, Jiancheng Weng, and Jian Rong. 2012. Simulation based mixed ETC/MTC freeway toll station capacity. 19th ITS World Congress.
- [14] Ding, Chuangxin. 2005. Study on capacity of high-grade highway toll station. Kunming University of Science and Technology.

Dear New Jersey Turnpike Administration:

The New Jersey Turnpike is about 200 miles long, the fifth longest of U.S. tollroads. The ETC utilization rate of toll booths of New Jersey is over 80%. In the future, the use of ETC is likely to increase. Meanwhile, progress in autonomous vehicles is accelerating; so in the next 20 years there may be a large number of autonomous vehicles on the Turnpike, which could pass through toll plazas at a faster speed. Toll plazas need to keep up with increased traffic flow and ensure the safety of vehicles.

We have come up with a cellular toll plaza design, whose conceptual model is shown below.

This design has four main advantages:

- 1. The cellular architecture can reduce plaza area about 50% compared to a traditional toll plaza design. In the figure below, the black part is our design for 8 toll booths, while the blue part is the traditional design.
- 2. Simulation results show that cellular toll plazas have better performance than traditional toll plazas, especially under heavy traffic, reducing average travel time through the plaza by about 50%.

It is our pleasure to offer policy recommendations regarding tolling on the New Jersey Turnpike. We have developed a new toll plaza design that can solve many problems caused by the increase of ETC service and autonomous cars.



Comparison of the footprints of the cellular toll plaza and a traditional toll plaza.

- 3. The Rule of 8: A plaza with 8 ETC toll booths is 8 times as fast as one with 8 MTC toll booths.
- 4. We simulated the performance of a cellular toll plaza under different traffic levels. The average transit time remains about 11 seconds for up to 2,000 vehicles/hr.

To give you a more intuitive view of the advantages of our cellular toll plaza, we have made a comparison diagram for you, shown above.

Furthermore, to help you understand the advantages of the cellular toll plaza with more ETC lanes, we offer a graph on p. 312 of our technical report, where the blue solid line stands for the performance of the traditional design and the red dashed line is for our design. You can see that with heavy traffic (but below the absolute capacity of the toll plaza), the time cost passing through the cellular toll plaza is lower than for the traditional one.

Thank you again for taking the time to read our suggestions. We sincerely hope that the cellular toll plaza can solve the congestion problem on the New Jersey Turnpike.

Sincerely,

The Toll Team



Team members Jiahua Zhang, Yang Liu, and Rui Liu, and advisor Hui Ji.